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HB-003-001515

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

May / June – 2017

Mathematics : BSMT-503(A)

(Discrete Mathematics & Complex Analysis-I)

(New Course)

Faculty Code : 003
Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Question-1 carries 20 marks of objective type questions.
(3) Each question has mentioned its marks on right hand side.

1 Answer all the following objective type questions : **20**

(1) Define transitive relation by giving an example of it.

(2) Is the relation $R=\{(1,1),(1,4),(1,4),(4,1),(4,4),(2,3),(3,2),(2,2),(3,5)\}$ on a set $X=\{1,2,3,4\}$ equivalent relation ?

(3) State least element of poset $(\{1,2,3,4,5,6\},1)$.

(4) Define Totally ordered set (chain). Is (S_{30},D), where $S_{30}=\{1,2,3,5,6,10,15,30\}$ a chain ? Why ?

(5) Give an example of a bounded lattice which is not a complemented lattice.

(6) To minimize the sum of product canonical form which map is useful ?

(7) Define complete lattice.

(8) Find out complement of 2 and 3 of the lattice $\left(S_{30}, *, \oplus\right)$.

(9) Define Atoms in Boolean Algebra.

(10) What are the atoms of Boolean algebra $\left(S_{30}, *, \oplus, ', 0, 1\right)$

(11) Evaluate $\lim_{z \to \infty} \frac{2z+3}{z+i}$

(12) Define analytic function.

(13) Check whether the function $f(z) = \bar{z}$ is analytic or not.

(14) State imaginary part of the function $(\sin x + i \cos x)^5$

(15) Define Harmonic function.

(16) State Cauchy-Riemann equations in Cartesian form.

(17) Find real part of the function $\frac{2+3i}{3-4i}$

(18) If C is an ellipse with centre at origin then find value of $\int_C \frac{\left(x^3+3\right)}{x} dz$.

(19) State harmonic conjugate of $u = \frac{1}{2} \log\left(x^2 + y^2\right)$

(20) Find singularities of $f(z) = \frac{2z}{z^2+1}$

2 (a) Attempt any **three** : **6**

(1) Show that $(z, \geq)$ is a poset.

(2) Draw the Hasse-diagram of $(P(A), C)$, where $A = \{a, b, c\}$.

(3) Find out complement of each element of the lattice $(S_{30}, *, \oplus)$.

(4) Show that (Z^{+}, D) is not a chain, where D means division.

(5) If (P, R) is a poset then show that $(P, \tilde{R})$ is also a poset.

(6) In usual notation prove that

$$[A(x)]^{C} = A - A(x) = A(x')$$

(b) Attempt any **three** : **9**

(1) Show that similarity of two matrices on the set of $n \times n$ matrices is an equivalence relation.

(2) Show that the relation $R = \{(x, y)/x - y$ is divisible by 3$\}$ on the set of integers z, is an equivalence relation.

(3) Let $P(A)$ be the power set of nonempty set A and for $X, Y \in P(A)$, $X * Y = X \cap Y$ and $X \oplus Y = X \cup Y$ then prove that $(P(A), *, \oplus)$ is a lattice.

(4) In a distributive lattice $(L, \leq)$ prove that $A \wedge B = a \wedge c$ and $a \vee b = a \vee c \Rightarrow b = c$.

(5) Define lattice and show that (S_6, D) is a lattice.

(6) Express Boolean expression $\propto(x_1, x_2, x_3) = x_1 \oplus x_2$ as the sum of product canonical form.

(c) Attempt any **two** : **10**

(1) If $(L, \leq)$ is a lattice then prove that

(i) $a \vee a = a$ and $a \wedge a = a \quad \forall\, a, b, c \in L$

(ii) $a \vee b = b \vee a$ and $a \wedge b = b \wedge a$

(iii) $a \wedge (b \vee c) = (a \vee b) \vee c$ and $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

(2) Prove that direct product of two lattices is also a lattice.

(3) State and prove stone's representation theorem.

(4) If $(B, *, \oplus, ', 0, 1)$ is a Boolean Algebra then prove that for any $x_1, x_2 \in B$

(i) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(5) State and prove D'-Morgan's law for the Boolean Algebra $(B, \phi, *, \oplus, ', 0, 1)$.

3 (a) Attempt any **three** : **6**

(1) Show that $\lim\limits_{z \to 0} \frac{\overline{z}}{z}$ does not exist.

(2) Define singular points and show that $f(z) = \overline{z}$ is not analytic function.

(3) Show that $f(z) = (3x + y) + i(3y - x)$ is entire function.

(4) Prove that $f(z) = \cos z$ is analytic function and find $f'(z)$.

(5) Find $\int\limits_c \frac{z^2}{z-1} dz, c : |z| = 2$

(6) State Cauchy's integral formula for derivative.

(b) Attempt any **three** : **9**

(1) Show that $f(z) = 1/z$ is an analytic function but not an entire function.

(2) If complex function $f(z) = u + iv$ and its conjugate $\overline{f}(z) = u - iv$ are analytic functions then prove that $f(z)$ is constant function.

(3) Show that an analytic function with constant modulus is constant.

(4) Find an analytic function $f(z)$ whose real part is $\cos x \cdot \cosh y$.

(5) Find $\int_c z^2 dz,$ c is a part of parabola $y = x^2$ from the point $z = 0$ to $z = 1 + i$.

(6) State and prove Liouville's theorem.

(c) Attempt any **two** : **10**

(1) If $w = f(z)$ is analytic function of u and v which are harmonic functions of r and θ, then prove that polar from of $f'(z)$ is

$$f'(z) = e^{-i\theta}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right) = \frac{r}{z}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)$$

(2) State and prove Morera's theorem.

(3) State and prove Cauchy's integral formula.

(4) If the complex function $f(z) = u + iv$ is an analytic function then prove that $\nabla^2 |f(z)|^2 = 4|f'(z)|^2$ and verify this result for $f(z) = z$.

(5) Obtain $C - R$ equations for an analytic function $f(z)$ in polar form.
